R2669

Sub. Code
511201

M.Sc. DEGREE EXAMINATION, APRIL – 2025

Second Semester

Mathematics

LINEAR ALGEBRA

(CBCS – 2022 onwards)

Time : 3 Hours Maximum : 75 Marks

Part A $(10 \times 1 = 10)$

Answer **all** the questions by choosing the correct option.

1. Any subset of V which contains more than n vectors is ————. (CO1, K1)

   (a) Linearly independent

   (b) Linearly dependent

   (c) finite

   (d) infinite

2. Let V is a finite-dimensional vector space, then any ——— bases of V have the same number of elements. (CO1, K1)

   (a) Four (b) Two

   (c) Three (d) Zero

3. If W is a k-dimensional subspace of an n-dimensional vector space V, then w is the intersection of ———— hyperspaces in V. (CO2, K1)

   (a) $(n-k)$ (b) (n)

   (c) $(n+k)$ (d) (k)

4. If V is a vector space over the field F and S is a subset of V, the annihilator of S is the set S^0 of linear functionals f on V such that ——————for every α in S. (CO2, K1)

(a) $f(\alpha)=1$ (b) $\alpha=0$

(c) $f(\alpha)=0$ (d) $f(\alpha+S)=0$

5. A polynomial f of degree n over a field F has at most —————— roots. (CO3, K1)

(a) $n-1$ (b) $n+1$

(c) 1 (d) n

6. The field F is called —————— if every prime polynomial over F has degree 1. (CO3, K1)

(a) algebraically open

(b) closed

(c) open

(d) algebraically closed

7. An $n\times n$ matrix A over a field F is called ———if $AA^t=I$. (CO4. K1)

(a) Orthogonal (b) Symmetric

(c) Skew-symmetric (d) Hermitian

8. Let T and U be a linear operators on the finite dimensional vector space over V. Then T is invertible if and only if ——————. (CO4, K1)

(a) $\det T=0$ (b) $\det T\neq 0$

(c) $\det T=1$ (d) $\det T\neq 1$

9. Let N be a linear operator on the vector space V. N is nilpotent if there is some positive integer r such that ————. (CO5, K1)

(a) $N^r = 1$ (b) $N^r \neq 1$

(c) $N^r + 1 = 0$ (d) $N^r = 0$

10. Every matrix A such that $A^2 = A$ is similar to a ————. (CO5, K1)

(a) Orthogonal matrix

(b) Diagonal matrix

(c) Symmetric matrix

(d) Hermitian matrix

Part B $(5 \times 5 = 25)$

Answer **all** questions not more than 500 words each.

11. (a) Show that the subspace spanned by a non-empty subset S of a vector space V is the set of all linear combinations of vectors in S. (CO1, K3)

Or

(b) Let R be a non zero row-reduced echelon matrix. Then prove that the non-zero row vectors of R from a basis for the row space of R. (CO1, K4)

12. (a) Let V be a finite-dimensional vector space over the field F, and let W be a subspace of V. Then Show that $\dim W + \dim W^0 = \dim V$. (CO2, K4)

Or

(b) If S is any subset of a finite-dimensional vector space V, then prove that $(S^0)^0$ is the subspace spanned by S. (CO2, K3)

13. (a) If F is a field, and M is any non-zero ideal in $F[x]$, then prove that there is a unique monic polynomial d in $F[x]$ such that M is the principal ideal generated by d. (CO3, K3)

Or

(b) Let p, f, and g be polynomials over the field F. Suppose that p is a prime polynomial and that p divides the product fg. Then prove that either p divides f or p divides g. (CO3, K3)

14. (a) Let K be a commutative ring with identity, and let A and B be $n \times n$ matrices over K. Then prove that $\det(AB) = (\det A)(\det B)$. (CO4, K4)

Or

(b) Let A be the (real) 3×3 matrix. (CO4, K3)

$$\begin{bmatrix} 3 & 1 & -1 \\ 2 & 2 & -1 \\ 2 & 2 & 0 \end{bmatrix}.$$

Find the Characteristic polynomial and Characteristic values for A.

15. (a) Let $\mathcal{F}$ be a commuting family of diagonalizable linear operators on the finite-dimensional vector space V. Then prove that there exists an ordered basis for V such that every operator in $\mathcal{F}$ is represented in that basis by a diagonal matrix.

(CO5, K3)

Or

(b) Let V be a finite-dimensional vector space over the Field F and let T be a linear operator on V. Then prove that T is diagonalizable if and only if the minimal polynomial for T has the form $p=(x-c_1)....(x-c_k)$ where $c_1,...,c_k$ are distinct elements of F. (CO5, K4)

Part C $(5 \times 8 = 40)$

Answer **all** the questions, not more than 1000 words each

16. (a) Let m and n be positive integers and let F be a field. Suppose W is a subspace of F^n and dim $W \leq m$. Then prove that there is precisely one $m \times n$ row-reduced echelon matrix over F which has W as its row space. (CO1, K6)

Or

(b) Consider the 5×5 matrix. (CO1, K5)

$$A=\begin{bmatrix} 1 & 2 & 0 & 3 & 0 \\ 1 & 2 & -1 & -1 & 0 \\ 0 & 0 & 1 & 4 & 0 \\ 2 & 4 & 1 & 10 & 1 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}.$$

(i) Find an invertible matrix P such that PA is a row-reduced echelon matrix R.

(ii) Find a basis for the row space W of A.

17. (a) Let V be an n-dimensional vector space over the field F, and let W be an m-dimensional vector space over F. Then find the space $L(V,W)$ is finite-dimensional and has dimension mn. (CO2, K6)

Or

(b) Let V and W be vector space over the field F, and let T be a linear transformation from V into W. The null space of T^t is the annihilator of the range of T. If V and W are finite-dimensional, then prove that the following: (CO2, K5)

(i) $rank(T') = rank(T)$

(ii) the range of T' is the annihilator of the null space of T.

18. (a) If F is a field, then prove that a non- scalar monic polynomial in $F[x]$ can be factored as a product of monic primes in $F[x]$ in one and, except for order, only one way. (CO3, K5)

Or

(b) State and prove Taylor's Formula. (CO3, K5)

19. (a) State and prove Cayley-Hamilton theorem. (CO4, K5)

Or

(b) Let T be the linear operator on R^3 which is represented in the standard ordered basis by the matrix (CO4, K5)

$$A = \begin{bmatrix} 5 & -6 & -6 \\ -1 & 4 & 2 \\ 3 & -6 & -4 \end{bmatrix}.$$

Compute the characteristic polynomial using various row and column operations.

20. (a) Let $\mathcal{F}$ be a commuting family of triangulable linear operators on V. Let W be a proper subspace of V which is invariant under $\mathcal{F}$. Then prove that there exist α vector a in V such that (CO5, K5)

(i) α is not in W;

(ii) for each T in $\mathcal{F}$, the vector $T\alpha$ is in the subspace spanned by α and W.

Or

(b) Let T be a linear operator on the finite-dimensional vector space V over the field F Suppose that the minimal polynomial for T decomposes over F into a product of linear polynomials Then prove that there is a diagonalizable operator D on V and a nilpotent operator N on V such that (CO5, K6)

(i) $T = D + N$,

(ii) $DN = ND$

R2670

Sub. Code
511202

M.Sc. DEGREE EXAMINATION, APRIL – 2025

Second Semester

Mathematics

REAL ANALYSIS - II

(CBCS – 2022 onwards)

Time : 3 Hours Maximum : 75 Marks

Part A $(10 \times 1 = 10)$

Answer **all** the questions by choosing the correct option.

1. If $f:[a, b] \to \mathbb{R}$, is continuous then which of the following is correct? (CO1, K5)

   (a) f is integrable on $\mathbb{R}$

   (b) f is integrable on $\mathbb{R} - [a, b]$

   (c) f must be integrable on $[a, b]$

   (d) f need not be integrable on $[a, b]$

2. Which of the following curves is rectifiable? (CO1, K2)

   (a) $y = |x|$ (b) $y = x^2$

   (c) $y = \sin\left(\frac{1}{x}\right)$ (d) $y = x^{-\frac{1}{2}}$

3. The sequence of real valued function $f_n(x) = x^n$, $x \in [0, 1] \cup \{2\}$ is (CO2, K3)

(a) Pointwise convergent

(b) Uniformly convergent

(c) Does not uniformly converges

(d) Divergent

4. Let $\{f_n\}$ be a sequence of functions defined on $[0, 1]$ that converges pointwise to a function *f*. Which of the following statements is not necessarily true? (CO2, K2)

(a) If each f_n is continuous, then *f* is continuous

(b) If $\{f_n\}$ converge uniformly to *f*, then *f* is continuous if each $\{f_n\}$ is continuous

(c) $\{f_n\}$ converges to *f* pointwise if and only if for each $x \in [0, 1]$, $\lim_{n \to \infty} f_n(x) = f(x)$

(d) For every $\in > 0$ and for each $x \in [0, 1]$, there exists an $N_0 \in \mathbb{N}$ such that $|f_n(x) - f(x)| \leq \in$ for all $n \geq N_0$.

5. Let $F = \{f_n\}$ be family of continuous function from a compact metric space to $\mathbb{R}$. Which of the following statements is true? (CO3, K3)

(a) If *F* is equicontinuous, then *F* is uniformly bounded

(b) If *F* is uniformly bounded, then *F* is equicontinuous

(c) If *F* is equicontinuous and pointwise bounded, then *F* is uniformly bounded

(d) If *F* is pointwise bounded and uniformly bounded, then *F* is equicontinuous

6. Let $\{f_n\}$ be sequence of functions defined on $[0,1]$, $f_n(x) = \dfrac{x^n}{1+x^n}$, then (CO3, K3)

(a) $\{f_n\}$ converges uniformly to f, where $f(x) = 0,\ x \in [0,1]$

(b) $\{f_n\}$ does not converges

(c) $\{f_n\}$ converges uniformly to f, where $f(x) = \begin{cases} 0, & x \in [0,1) \\ \frac{1}{2}, & x = 1 \end{cases}$

(d) $\{f_n\}$ converges uniformly to f, where $f(x) = \begin{cases} 0, & x \in [0,1) \\ 1, & x = 1 \end{cases}$

7. Suppose the series $\sum_{n=0}^{\infty} C_n x^n$ converges for $|x| \le \mathbb{R}$, define $f(x) = \sum_{n=0}^{\infty} C_n x^n$ where $|x| \le \mathbb{R}$, then f is (CO4, K3)

(a) continuous but not differentiable in $\mathbb{R}$

(b) not continuous and not differentiable in $\mathbb{R}$

(c) continuous and differentiable in $\mathbb{R}$

(d) not continuous but differentiable in $\mathbb{R}$

8. Evaluate $\dfrac{\log x + \log x^4 + \log x^9 \ldots + \log x^{n^2}}{\log x + \log x^2 + \log x^3 \ldots + \log x^n}$ (CO4, K5)

(a) $\dfrac{2n+1}{3}$

(b) $\dfrac{2n-1}{3}$

(c) $\dfrac{3(n+2)}{2}$

(d) None of these

9. What is the value of $\Gamma(1)$? (CO5, K3)

(a) 0 (b) 1

(c) e (d) π

10. What is the relation between Gamma function and factorial? (CO5, K2)

(a) $\Gamma(n) = (n-1)!$ (b) $\Gamma(n) = (n+1)!$

(c) $\Gamma(n) = (n)!$ (d) $\Gamma(n) = \frac{1}{n}!$

Part B (5 × 5 = 25)

Answer **all** the questions not more than 500 words each.

11. (a) Prove that if P^* is a refinement of P, then $L(P, f, \alpha) \leq L(P^*, f, \alpha)$ and $U(P^*, f, \alpha) \leq U(p, f, \alpha)$. (CO1, K5)

Or

(b) State and prove fundamental theorem of calculus. (CO1, K5)

12. (a) Prove that if f is a sequence of continuous functions on E, and if $f_n \to f$ uniformly on E, then f is continuous on E. (CO2, K5)

Or

(b) State and prove the Cauchy criterion for uniform convergence. (CO2, K5)

13. (a) If K is compact metric space, if $f_n \in \mathcal{C}(K)$ for n = 1, 2, 3,... and $\{f_n\}$ converges uniformly on K, then prove that $\{f_n\}$ is equicontinuous on K.

(CO3, K3)

Or

(b) Prove that if $\{f_n\}$ is pointwise bounded sequence of complex functions on a countable set E, then $\{f_n\}$ has a subsequences $\{f_{n_k}\}$ such that converges for every $x \in E$. (CO3, K3)

14. (a) Prove that the exponential function $E(z)$ has the following properties : (CO4, K2)

(i) $E(z+w) = E(z)E(w)$ for all $z, w \in \mathcal{C}$

(ii) $E(z) = 0$ for all $z \in \mathcal{C}$

(iii) The function E is differentiable and $E'(z) = E(z)$ for all $z, w \in \mathcal{C}$.

Or

(b) Prove the following : (CO4, K2)

(i) The function E is periodic, with period $2\pi i$

(ii) The function C and S are periodic, with period 2π .

(iii) If $0 \le t \le 2\pi$, then $E(it) \ne 1$.

15. (a) Prove that if f is continuous with period 2π and if $\in \geq 0$, then there is a trigonometric polynomial P such that $|P(x) - f(x)| \leq \in$ for all real x. (CO5, K4)

Or

(b) Let f be a positive function on $(0, \infty)$ such that

(i) $f(x+1) = xf(x)$ for all x (CO5, K6)

(ii) $f(1) = 1$

(iii) $\log f(x)$ is convex.

Then prove that $f(x) = \Gamma(x)$ for all x.

Part C (5 × 8 = 40)

Answer **all** the questions not more than 1000 words each.

16. (a) (i) Prove that if f is bounded on $[a,b]$, f has only finitely many points of discontinuity on $[a,b]$, and α is continuous at every point at which f is discontinuous. Then $f \in \mathcal{R}(\alpha)$.

(ii) Prove that if f is continuous on $[a,b]$, then $f \in \mathcal{R}(\alpha)$ on $[a,b]$. (CO1, K5)

Or

(b) If γ' is continuous on $[a,b]$, then prove that γ is rectifiable and $\Lambda(\gamma) = \int_a^b |\gamma'(t)| dt$. (CO1, K5)

17. (a) Show that $(\mathcal{C}(X), \|\cdot\|)$ is a complete metric space. (CO2, K2)

Or

(b) Suppose K is compact, and (CO2, K5)

(i) f_n is a sequence of continuous functions on K

(ii) f_n converges pointwise to a continuous function f on K,

(iii) $f_n(x) \geq f_{n+1}(x)$ for all $x \in K, n = 1, 2, 3 ....$

Then prove that $f_n \to f$ uniformly on K

18. (a) State and prove Stone-Weierstrass theorem. (CO2, K5)

Or

(b) Suppose $\{f_n\}$ is a sequence of functions, differentiable on $[a,b]$ and such that $\{f_n(x_0)\}$ converges for some point x_0 on $[a,b]$. If $\{f_n'\}$ converges uniformly on $[a,b]$, then prove that $\{f_n\}$ converges uniformly on $[a,b]$ to a function f, and $f'(x) = \lim_{n\to\infty} f_n'(x), (a \leq x \leq b)$. (CO3, K4)

19. (a) Prove that e^x defined on $\mathbb{R}$ has the following properties: (CO4, K4)

(i) e^x is strictly increasing on $\mathbb{R}$

(ii) $(e^x)' = e^x$

(iii) e^x is continuous and differentiable for all x

(iv) $e^{x+y} = e^x e^y$

(v) $\lim_{x\to+\infty} x^n e^{-x} = 0$, for every n

(vi) $e^x \to +\infty$ as $x \to +\infty$, $e^x \to 0$ as $x \to -\infty$

Or

(b) (i) Suppose $\sum C_n$ converges. Put $f(x) = \sum_{n=0}^{\infty} C_n X^n \,(-1 < x < 1)$. Then prove that $\lim_{x \to 1} f(x) = \sum_{n=0}^{\infty} C_n$.

(ii) Given a double sequence (a_{ij}), $i = 1, 2, 3, \ldots$, $j = 1, 2, 3, \ldots$, suppose that $\sum_{j=1}^{\infty} |a_{ij}| = b_i$ $(i = 1, 2, 3, \ldots)$ and $\sum b_i$ converges. Then prove that $\sum_{i=1}^{\infty} \sum_{j=1}^{\infty} a_{ij} = \sum_{j=1}^{\infty} \sum_{i=1}^{\infty} a_{ij}$. (CO4, K5)

20. (a) State and prove Parseval's theorem. (CO5, K5)

Or

(b) State and prove Bessal's inequality. (CO5, K5)

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R2671

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511203

M.Sc. DEGREE EXAMINATION, APRIL – 2025

Second Semester

Mathematics

COMPLEX ANALYSIS

(CBCS – 2022 onwards)

Time : 3 Hours Maximum : 75 Marks

Part A ($10 \times 1 = 10$)

Answer **all** the questions by choosing the correct option.

1. Which of the following function $f(z)$, of the complex variable z, is not analytic at all the points of the complex plane? (CO1, K1)

   (a) $f(z) = z^2$ (b) $f(z) = e^z$

   (c) $f(z) = \sin z$ (d) $f(z) = \log z$

2. Which of the following is true? (CO1, K1)

   (a) Differentiability does not imply continuity

   (b) Differentiability implies continuity

   (c) Continuity implies differentiability

   (d) There is no relation between continuity and differentiability

3. If $P(z)$ is a polynomial of degree $n(n \geq 1)$ then it has (CO2, K1)

   (a) $n+1$ zeros (b) n zeros

   (c) $n-1$ zeros (d) no zeros

4. The integral of the function $\int_C e^z \cos z \, dz$ where C is the unit circle is (CO2, K1)

(a) $\frac{\pi}{2}(3+2i)$ (b) $\pi(3+2i)$

(c) $\frac{\pi}{3}(3+2i)$ (d) $\frac{\pi}{2}(2+3i)$

5. The power series representation of $\frac{1}{1-z}$ in non-negative powers of z is (CO3, K1)

(a) $1+z+z^2+z^3+\cdots$

(b) $1-z+z^2-z^3+\cdots$

(c) $1+z+z^3+z^5+\cdots$

(d) $1-z+z^3-z^5+\cdots$

6. The order of the zeros of the function $\frac{\sin z}{z+4}$ is (CO3, K1)

(a) 1 (b) 2

(c) 3 (d) 4

7. Residue of the function $\cot z$ at the singular points is (CO4, K1)

(a) –1 (b) 1

(c) –2 (d) 0

8. Singularities of rational functions are (CO4, K1)

(a) Poles (b) Essential

(c) Non-isolated (d) Removable

9. In the Laurent expansion of $f(z) = \dfrac{1}{(z-1)(z-2)}$ valid in the region $1 < |z| < 2$, the coefficient of $\dfrac{1}{z^2}$ is (CO5, K2)

(a) 0 (b) $\dfrac{1}{2}$

(c) 1 (d) –1

10. The Laurent series expansion of the function $f(z) = \dfrac{1}{e^z - 1}$ valid in the region $0 < |z| < 2$ is given by (CO5, K2)

(a) $f(z) = \dfrac{1}{z} - \dfrac{1}{2} + \dfrac{1}{3}z - \dfrac{1}{120}z^3 + \cdots$

(b) $f(z) = \dfrac{1}{z} + \dfrac{1}{2} - \dfrac{1}{3}z + \dfrac{1}{120}z^3 + \cdots$

(c) $f(z) = \dfrac{1}{z} - \dfrac{1}{2} + \dfrac{1}{12}z - \dfrac{1}{720}z^3 + \cdots$

(d) $f(z) = \dfrac{1}{z} - \dfrac{1}{2} + \dfrac{1}{12}z - \dfrac{1}{120}z^3 + \cdots$

Part B (5 × 5 = 25)

Answer **all** the questions not more than 500 words each.

11. (a) If all zeros of a polynomial $P(z)$ lie in a half plane, then prove that all zeros of the derivative $p'(z)$ lie in the same half plane. (CO1, K2)

Or

(b) Derive the complex form of the Cauchy-Riemann equations. (CO1, K2)

12. (a) State and prove Cauchy's representation formula. (CO2, K5)

Or

(b) Suppose that $f(z)$ is analytic on a closed curve γ (i.e., f is analytic in a region that contains γ). Show that $\int_{\gamma} \overline{f(z)} f'(z) dz$ is purely imaginary. (The continuity of $f'(z)$ is taken for granted.) (CO2, K2)

13. (a) Prove that an analytic function comes arbitrarily close to any complex value in every neighborhood of an essential singularity. (CO3, K5)

Or

(b) State and prove Taylor's theorem. (CO3, K5)

14. (a) Evaluate the following integrals by the method of residues (CO4, K6)

(i) $\int_{0}^{\infty} \frac{x^2 dx}{x^4 + 5x^2 + 6}$

(ii) $\int_{0}^{\infty} \frac{x^2 dx}{(x^2 + a^2)^3}$, a real

Or

(b) State and prove maximum principle for harmonic function. (CO4, K5)

15. (a) Define an entire function with an example. Also prove that every function which is meromorphic in the whole plane is the quotient of two entire functions. (CO5, K2)

Or

(b) In the functions $f_n(z)$ are analytic and $\neq 0$ in a region Ω, and if $f_n(z)$ convergences to $f(z)$, uniformly on every compact subset of Ω, then prove $f(z)$ is either identically zero or never equal to zero in Ω. (CO5, K5)

Part C (5 × 8 = 40)

Answer **all** the questions not more than 1000 words each.

16. (a) State and prove Abel's limit theorem. (CO1, K5)

Or

(b) Prove that a sequence is convergent if and only if it is a Cauchy sequence. (CO1, K6)

17. (a) Suppose that $\varphi(\zeta)$ is continuous on the arc γ. Then prove that function $F(z) = \int_{\gamma} \frac{\varphi(\zeta)\,d\zeta}{(\zeta - z)^n}$ is analytic in each of the regions determined by γ, and its derivative is $F_n'(z) = nF_{n+1}(z)$. (CO2, K5)

Or

(b) Let $f(z)$ be analytic an the set R' obtained from a rectangle R by omitting a finite number of interior points ς_j. If it is true that $\lim_{z \to \varsigma_j} (z - \varsigma_j) f(z) = 0$ for all j, then prove that $\int_{\partial R} f(z)\,dz = 0$. (CO2, K5)

18. (a) If $f(z)$ is defined and continuous on a closed bounded set E and analytic on the interior of E, then prove that the maximum of $|f(z)|$ on E is assumed on the boundary of E. (CO3, K6)

Or

(b) Suppose that $f(z)$ is analytic in the region Ω' obtained by ommitting a point a from a region Ω. Prove that a necessary and sufficient condition that there exist an analytic function in Ω which coincides with $f(z)$ in Ω' is that $\lim_{z \to a}(z-a)f(z)=0$. The extended function is uniquely determined. (CO3, K5)

19. (a) Let γ be homologous to zero in Ω and such that $n(\gamma, z)$ is either 0 or 1 for every point z not on γ. Suppose that $f(z)$ and $g(z)$ are analytic in Ω and satisfy the inequality $|f(z)-g(z)|<|f(z)|$ on γ. Then prove that $f(z)$ and $g(z)$ have the same number of zeros enclosed by γ. (CO4, K6)

Or

(b) If $f(z)$ is meromorphic in Ω with the zeros a_j and the poles b_k, then prove that $\frac{1}{2\pi i}\int_{\gamma} \frac{f'(z)}{f(z)} dz = \sum_j n(\gamma, a_j) - \sum_k n(\gamma, b_k)$ for every cycle γ which is homologous to zero in Ω and does not pass through any of the zeros or poles. (CO4, K5)

20. (a) Suppose that $f_n(z)$ is analytic in the region Ω_n, and that the sequence $\{f_n(z)\}$ converges to a limit function $f(z)$ in a region Ω, uniformly on every compact subset of Ω. Then prove that $f(z)$ is analytic in Ω. Moreover, $f_n'(z)$ converges uniformly to $f'(z)$ on every compact subset of Ω'. (CO5, K6)

Or

(b) If $f(z)$ is analytic in the region Ω, containing z_0, then show that the representation (CO5, K5)

$$f(z) = f(z_0) + \frac{f'(z_0)}{1!}(z - z_0) + \cdots + \frac{f^{(n)}(z_0)}{n!}\left(z - z_0{}^n\right)$$

is valid in the largest open disk of center z_0 contained in Ω.

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R2672

Sub. Code
511204

M.Sc. DEGREE EXAMINATION, APRIL – 2025

Second Semester

Mathematics

PARTIAL DIFFERENTIAL EQUATION

(CBCS – 2022 onwards)

Time : 3 Hours Maximum : 75 Marks

Part A $(10 \times 1 = 10)$

Answer **all** the questions by choosing the correct option.

1. The Pfaffian equation is typically used to describe which of the following? (CO1, K1)

   (a) A second-order system of partial differential equations

   (b) A first-order system of differential equations.

   (c) A set of boundary conditions for partial differential equations.

   (d) A system of ordinary differential equations.

2. What is the general form of a Pfaffian differential equation in three variables x, y, and z? (CO1, K2)

   (a) $P(x,y,z)dx + Q(x,y,z)dy + R(x,y,z)dz = 0$

   (b) $P(x,y,z)dx + Q(x,y,z)dy + R(x,y,z) = 0$

   (c) $P(x,y,z)dx + Q(x,y,z)dy = 0$

   (d) $P(x,y,z)dx + Q(x,y,z)dy + R(x,y,d)z = 1$

3. Which of the following is a general form of a first-order partial differential equation? (CO2, K2)

(a) $\frac{\partial u}{\partial x}+\frac{P(x,y)\partial u}{\partial y}=Q(x,y)$

(b) $\frac{\partial u}{\partial x}+\frac{\partial u}{\partial y}=0$

(c) $\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}=0$

(d) $\frac{\partial^2 u}{\partial x^2}=P(x,y)\frac{\partial u}{\partial y}$

4. The Cauchy method of characteristics is primarily used for solving which type of partial differential equation? (CO2, K1)

(a) Second-order linear PDEs.

(b) First-order linear and nonlinear PDEs.

(c) Second-order nonlinear PDEs.

(d) Elliptic equations.

5. For the Jacobi method to converge, the coefficient matrix A must be: (CO3, K1)

(a) Symmetric positive definite

(b) Non-singular

(c) Sparse

(d) Diagonally dominant or symmetric positive definite

6. In the Jacobi method, the system $Ax=b$ is iteratively solved using: (CO3, K1)

(a) The inverse of A

(b) The whole matrix A in each iteration

(c) An approximation of A for each iteration

(d) Only the diagonal elements of A

7. The method of separation of variables is primarily used to solve: (CO4, K1)

   (a) Nonlinear differential equations

   (b) First-order partial differential equations

   (c) Linear partial differential equations

   (d) Ordinary differential equations

8. Which of the following is an example of an integral transform? (CO4, K1)

   (a) Fourier Transform

   (b) Laplace Transform

   (c) Z-Transform

   (d) All of the above

9. Which of the following is a common boundary condition for the heat equation in a boundary value problem? (CO5, K1)

   (a) $u(x, 0) = 0$

   (b) $u(0, t) = (L, t) = 0$

   (c) $u(x, t) = T_0$

   (d) $T\frac{\partial u}{\partial x} = 0$

10. The general solution to the one-dimensional wave equation is: (CO5, K1)

   (a) $u(x,t) - f(x+ct) + g(x-ct)$

   (b) $u(x,t) - f(x-ct) + g(x-ct)$

   (c) $u(x,t) = f(x) + g(t)$

   (d) $u(x,t) = f(x) \cdot g(t)$

Part B (5 × 5 = 25)

Answer **all** the questions not more than 500 words each.

11. (a) Find the orthogonal trajectories on the cone $x^2+y^2=z^2\tan^2\alpha$ of the intersection with the family of planes parallel to $z=0$. (CO1, K3)

Or

(b) Solve the equation $x(y^2-a^2)dx+y(x^2-z^2)dy$ $-z(y^2-a^2)dz=0$ is integrable. (CO1, K4)

12. (a) Find the integral surfaces of the linear partial differential equation $x(y^2+z)p-y(x^2+z)q=$ $(x^2-y^2)z$ which contains the straight line $x+y=0$, $z=1$. (CO2, K4)

Or

(b) Find the general solution of the differential equation $x^2\dfrac{\partial^2 z}{\partial x}+y^2\dfrac{\partial^2 z}{\partial y}=(x+y)z$. (CO2, K3)

13. (a) Explain the Charpit' s method. (CO3, K4)

Or

(b) Solve the equation $xp=yq, z(xp+yq)=2xy$ are compatible. (CO3, K3)

14. (a) If $u_1, u_2, ,..., u_n$ are solutions of the homogeneous linear partial differential equation $F(D,D')z=0$, then prove that $\sum_{r=1}^{n} c_r\, u_r$ where the c_r are arbitrary constant. (CO4, K3)

Or

(b) Find the particular integral of the equation $(D^2 - D')z = e^{x+y}$. (CO4, K4)

15. (a) Determine the temperature $\theta(\rho, t)$ in the infinite cylinder $0 \leq \rho \leq a$, When the initial temperature is $\theta(\rho, 0) = f(p)$ and the surface $\rho = a$ is maintained at zero temperature. (CO5, K3)

Or

(b) Explain the separation of variables. (CO5, K4)

Part C (5 × 8 = 40)

Answer **all** the questions, not more than 1000 words each

16. (a) Prove that the necessary and sufficient condition of Pfaffian differential equation $X \cdot dr = 0$ should be integrable is that $X \cdot curl\, X = 0$. (CO1, K5)

Or

(b) If X is a vector such that $X \cdot curl\, X = 0$ and μ is an arbitrary function of x, y, z then prove that $(\mu X) \cdot curl(\mu X) = 0$. (CO1, K5)

17. (a) Prove that the characteristic strip contains at least one integral element of $F(x, y, z, p, q) = 0$ it is an equation strip of the equation $F(x, y, z, z_x, z_y) = 0$.

(CO2, K6)

Or

(b) Find the solution of the equation $z = \frac{1}{2}(p^2 + q^2) + (p - x)(q - y)$ which passes through the x-axis. (CO2, K6)

18. (a) Find a complete integral of the equation $p^2x+q^2y=z$. (CO3, K6)

Or

(b) Show that the only integral surface of the equation $2q(z-px-qy)=1+q^2$ which is circumscribed about the paraboloid $2x=y^2+z^2$ is the enveloping cylinder which touchers tit along its section by the plane $y+1=0$. (CO3, K5)

19. (a) Prove that the operator $F(D,D')$ is reducible, the order in which the linear factors occur is unimportant. (CO4, K5)

Or

(b) Prove that $F(D,D')\{e^{ax+by}\phi(x,y)\}=e^{ax+by}$ $F(D+a,D'+b)\phi(x,y)$. (CO4, K5)

20. (a) The point of trisection of a string are pulled aside through a distance $\in$ on opposite sides of the position of equilibrium and the string is released from reset. Drive an expression for the displacement of the string at any subsequent time and show that the midpoint of the string always remains at rest. (CO5, K6)

Or

(b) The faces $x=0, x=a$ of an infinite slab are maintained at zero temperature. The initial distribution of temperature in the slab is described by the equation $\theta=f(x)\,(0\le x\le a)$. Determine the temperature at a subsequent time t. (CO5, K5)

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R2673

Sub. Code
511510

M.Sc. DEGREE EXAMINATION, APRIL – 2025

Second Semester

Mathematics

Elective – INTRODUCTION TO PYTHON PROGRAMMING

(CBCS – 2022 onwards)

Time : 3 Hours Maximum : 75 Marks

Part A $(10 \times 1 = 10)$

Answer **all** the following objective questions by choosing the correct option.

1. A group of people maintain exclusive control over the source code of software. Such software is called ———— (CO1, K2)

   (a) Freeware (b) Shareware

   (c) Proprietary (d) Adware

2. Which Python library is popularly referred to as the HTTP library written for humans. (CO1, K1)

   (a) Design (b) Development

   (c) Testing (d) Planning

3. Which of the following is not a keyword in Python language? (CO2, K1)

   (a) Val (b) Raise

   (c) Try (d) With

4. Amongst which of the following is / are the method of list? (CO2, K1)

(a) append ( ) (b) extend()

(c) insert() (d) All of the mentioned above

5. The output to execute string.ascii_letters can also be obtained from? (CO3, K1)

(a) Character

(b) ascii_lowercase_string.digits

(c) lowercase_string.upercase

(d) ascii_lowercase+string.ascii_upercase

6. Which of the following declarations is incorrect in python language? (CO3, K1)

(a) xyzp = 5,000,000

(b) xy z p = 5000 6000 7000 8000

(c) x,y,z,p = 5000, 6000, 7000, 8000

(d) x_y_z_p = 5,000,000

7. The basic ndarray is created using? (CO4, K2)

(a) numpy.array(object, dtype = None, copy = True, subok = False, ndmin = 0)

(b) numpy.array(obiect, dtype = None, copy = True, order = None, subok = False, ndmin = 0)

(c) numpy_array(obiect, dtype = None, copy = True, order = None, subok = False, ndmin =0)

(d) numpy.array(object, dtype = None, copy = True, order = None.ndmin=0)

8. What is the name of the operator **in Pyton? (CO4, K2)

(a) Exponentiation

(b) Modulus

(c) Floor division

(d) None of the mentioned above

9. The % operator returns the ―――――― (C

(a) Quotient

(b) Divisor

(c) Remainder

(d) None of the mentioned above

10. Python supports the creation of anonymous functions at runtime, using a construct called ―――――― (CO5, K2)

(a) pi (b) anonymous

(c) lambda (d) beta

Part B $(5 \times 5 = 25)$

Answer **all** the questions in not more than 500 words each.

11. (a) Write a program to convert kilogram into pound. (CO1, K3)

Or

(b) Write a program that calculates the number of seconds in a day. (CO1, K2)

12. (a) Differentiate between user-defined function and built-in functions. (CO2, K2)

Or

(b) Explain the built-in functions with examples in Python. (CO2, K2)

13. (a) Differentiate the syntax of if...else and if...elif..else with an example. (CO3, K2)

Or

(b) Explain the syntax of while loop with an example. (CO3, K3)

14. (a) Write a sort note on data types in Python. (CO4, K2)

Or

(b) Write a program to convert kilogram into pound. (CO4, K3)

15. (a) Write a program in Python to find GCD of two or more integers. (CO5, K4)

Or

(b) Write a program in Python to find prime numbers for the given integers. (CO5, K3)

Part C $(5 \times 8 = 40)$

Answer **all** the questions in not more than 1000 words each.

16. (a) Write a program to read two integers and perform arithmetic operations on them (addition, subtraction, multiplication and division). (CO1, K3)

Or

(b) Briefly explain binary left shift and binary right shift operators with examples. (CO1, K2)

17. (a) Write a program using functions to print harmonic progression series and its sum till N terms. (CO2, K2)

Or

(b) Write a program using functions to do the following tasks: (CO2, K2)

(i) Convert milliseconds to hours, minutes and seconds.

(ii) Compute a sales commission, given the sales amount and the commission rate.

(iii) Convert Celsius to Fahrenheit.

(iv) Compute the monthly payment, given the loan amount, number of years and the annual interest rate.

18. (a) Write a program that uses a while loop to add up all the even numbers between 100 and 200. (CO3, K2)

Or

(b) Write a program to print the sum of the following series (CO3, K3)

(i) $1+\frac{1}{2}+\frac{1}{3}+....+\frac{1}{n}$

(ii) $\frac{1}{1}+\frac{22}{2}+\frac{33}{3}+...+\frac{n^n}{n}$.

19. (a) A car starts from a stoplight and is traveling with a velocity of 10 m/sec east in 20 seconds. Write a program to find the acceleration of the car.(CO4, K2)

Or

(b) Write a program to convert temperature from centigrade (read it as float value) to Fahrenheit. (CO4, K3)

20. (a) Write a program in Python to find the product of the two matrices. (CO5, K4)

Or

(b) Write a program in Python to find the mean, median, mode and standard deviation for the given integers. (CO5, K3)

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R2674

Sub. Code
511401

M.Sc. DEGREE EXAMINATION, APRIL – 2025

Fourth Semester

Mathematics

FUNCTIONAL ANALYSIS

(CBCS – 2022 onwards)

Time : 3 Hours Maximum : 75 Marks

Part A $(10 \times 1 = 10)$

Answer **all** the questions by choosing the correct option.

1. What defines compactness in a metric space? (CO1, K1)

   (a) Every sequence has a convergent subsequence

   (b) The set is bounded and closed

   (c) The set is bounded

   (d) The set contains infinitely many points

2. What is a Schauder basis in a Banach space? (CO1, K1)

   (a) A set of linearly dependent vectors

   (b) A complete orthonormal system

   (c) A set allowing unique representation of elements

   (d) A finite-dimensional spanning set

3. Which set is not closed in l^∞ or l^2? (CO2, K2)

(a) Bounded sequences in l^∞

(b) Convergent sequences in l^2

(c) Sequences with finite sum of squares in l^2

(d) Sequences with limit points

4. Which is true about orthonormal sets? (CO2, K1)

(a) All orthonormal sets are orthogonal

(b) Orthonormal sets are always linearly dependent

(c) An orthonormal set is linearly independent

(d) All orthonormal sets are orthogonal and dependent

5. How are inner product and norm related in a Hilbert space? (CO3, K2)

(a) The norm defines the inner product

(b) The inner product defines the norm

(c) They are independent

(d) The norm is always larger than the inner product

6. Which is true about unitary operators? (C03, K1)

(a) All unitary operators are self-adjoint

(b) Unitary operators are not always self-adjoint

(c) Only projection unitary operators are self-adjoint

(d) A unitary operator is self-adjoint if diagonal

7. When is a space reflexive? (CO4, K1)

(a) If it is isomorphic to its dual space

(b) If it is finite-dimensional

(c) If all bounded functionals are continuous

(d) If it is closed and convex

8. What defines an adjoint operator? (CO4, K1)

(a) The adjoint commutes with the operator

(b) The adjoint is the inverse of the operator

(c) $\langle Tx, y\rangle = \langle x, T^{*}y\rangle$

(d) The adjoint is the transpose of the operator matrix

9. What is the difference between weak and strong convergence? (CO5, K2)

(a) Weak convergence requires norm convergence

(b) Weak convergence is for functionals, strong is for sequences

(c) Strong convergence is in norm, weak convergence is for functionals

(d) Weak and strong convergence are the same in Banach spaces

10. How do sequences of operators converge? (CO5, K1)

(a) Strong convergence means pointwise convergence

(b) Weak convergence requires boundedness of operators

(c) Norm convergence occurs when the operator norm converges

(d) Weak convergence happens when functionals converge

Part B $(5 \times 5 = 25)$

Answer **all** the questions not more than 500 words each.

11. (a) Show that a subspace Y of a Banach space X is complete if and only if the set Y is closed in X. (CO1, K2)

Or

(b) State and prove Riesz's Lemma. (CO1, K3)

12. (a) Prove that an orthonormal set is linearly independent. (CO2, K2)

Or

(b) If Y is a closed subspace of a Hubert space H, then show that $Y = Y^{\perp\perp}$. (CO2, K2)

13. (a) Let the operator $U : H \to H$ and $V : H \to H$ be unitary; H is a Hilbert space. Then prove that: (CO3, K3)

(i) $U^{-1}(=U^*)$ is unitary

(ii) UV is unitary;

(iii) U is normal.

Or

(b) If S and T are normal linear operators satisfying $ST^* = T^*S$ and $TS^* = S^*T$, then prove that their sum $S+T$ and product ST are normal. (CO3, K2)

14. (a) State and prove Zorn's lemma. (CO4, K3)

Or

(b) State and prove uniform boundedness theorem. (CO4, K3)

15. (a) State and prove closed graph theorem. (CO5, K3)

Or

(b) If X be a finite dimensional normed space, then show that weak convergence of a sequence implies strong convergence. (CO5, K2)

Part C $(5 \times 8 = 40)$

Answer **all** the questions not more than 1000 words each.

16. (a) If Y is a Banach space, then prove that $B(X, Y)$ is a Banach space. (CO1, K3)

Or

(b) Prove that the dual space of l^p is l^q; here, $1 < p < +\infty$ and q is the conjugate of p, that is $1/p + 1/q = 1$. (CO1, K3)

17. (a) Let X be an inner product space and $M \neq \phi$ a convex subset which is complete (in the metric induced by the inner product). Then prove that for every given $x \in X$ there exists a unique $y \in M$ such that $\delta = \inf_{\tilde{y} \in M} \|x - \tilde{y}\| = \|x - y\|$. (CO2, K3)

Or

(b) Prove that the two Hilbert spaces H and $\tilde{H}$, both real or both complex, are isomorphic if and only if they have the same Hilbert dimension. (CO2, K3)

18. (a) Prove that the Hilbert-adjoint operator T^* of a bounded linear operator $T : H_1 \to H_2$ exists, is unique and is a bounded linear operator with norm $\|T^*\| = \|T\|$. (CO3, K3)

Or

(b) State and prove Riesz representation Theorem. (CO3, K4)

19. (a) State and prove Hahn-Banach theorem. (CO4, K4)

Or

(b) State and Prove Baire's Category theorem. (CO4, K4)

20. (a) State and prove open mapping theorem. (C05, K4)

Or

(b) Let (x_n) be a sequence in a normed space X. Then prove that (CO5, K3)

(i) Strong convergence implies weak convergence with the same limit.

(ii) The converse of (*i*) is not generally true.

(iii) If dim $X < \infty$, then weak convergence implies strong convergence.

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R2675

Sub. Code
511402

M.Sc. DEGREE EXAMINATION, APRIL – 2025

Fourth Semester

Mathematics

PROBABILITY AND STATISTICS

(CBCS – 2022 onwards)

Time : 3 Hours   Maximum : 75 Marks

Part A $(10 \times 1 = 10)$

Answer **all** the questions by choosing the correct option.

1. $P(B \mid A) =$.   (CO1, K1)

   (a) $P(A \cap B)/P(B)$ (b) $P(A \cap B)/P(A)$

   (c) $P(A \cup B)/P(B)$ (d) $P(A \cup B)/P(A)$

2. Let X be a real-valued random variable with $E[X]$ and $E[X^2]$ denoting the mean values of X and X^2, respectively. The relation which always holds is.   (CO1, K1)

   (a) $\left(E[X]^2\right) > E\left(X^2\right)$ (b) $E\left(X^2\right) \geq \left(E[X]\right)^2$

   (c) $E\left[X^2\right] = (E([X])^2$ (d) $E\left[X^2\right] > \left(E[X]\right)^2$

3. ____________is depends on the outcome of the experiment.   (CO2, K1)

   (a) random vector   (b) random scale

   (c) random sample   (d) random mean

4. Select the wrong one? (CO2, K1)

(a) $E(X+Y)=E(X)+E(Y)$

(b) $E(X+bY)=E(X)+bE(Y)$

(c) $E(X+bY)=bE(X)+E(Y)$

(d) $E(aX)=aE(X)$

5. Consider a Poisson distribution for the tossing of a biased coin. The mean for this distribution is 4μ. The standard deviation for this distribution is given by. (CO3, K1)

(a) $\sqrt{\mu}$ (b) μ^2

(c) 2μ (d) $1/2\mu$

6. Putting $\alpha = 1$ in Gamma distribution results in.(CO3, K1)

(a) Exponential Distribution

(b) Normal Distribution

(c) Poisson Distribution

(d) Binomial Distribution

7. _____________ is the branch of mathematics for collecting, analysing and interpreting data. (CO4, K1)

(a) probability (b) random variable

(c) statics (d) statistic

8. If the between-groups variance and within-groups variance are 250 and 100 respectively and there are 5 groups with 10 observations each, what is the approximate t-value? (CO4, K1)

(a) 2.5 (b) 3.54

(c) 5 (d) 10

9. Which of the following best defines an estimator? (CO5, K1)

   (a) A person who hesitates or wavers in making decisions

   (b) An approximate calculation of a quantity based on observed data

   (c) A rule or formula used to calculate an estimate of a given quantity based on observed data

   (d) None of the above

10. Which of the following statements regarding convergence in distribution is true? (CO5, K1)

   (a) It guarantees convergence of sample means to population means

   (b) It implies convergence of random variables to a fixed value with high probability

   (c) It is equivalent to almost sure convergence

   (d) It describes the convergence of the cumulative distribution functions of random variables

Part B (5 × 5 = 25)

Answer **all** questions not more than 500 words each.

11. (a) State and prove Bayes' theorem. (CO1, K4)

Or

   (b) For any random variable, show that $P[X = x] = F_X(x) - F_X(x-)$ for all $x \in R$, where $F_X(x-) = \lim_{z\ x} F_X(z)$. (CO1, K3)

12. (a) Let X_1 and X_2 have the joint pdf

$$f(x_1, x_2) = \begin{cases} x_1 + x_2 & 0 < x_1 < 1,\ 0 < x_2 < 1 \\ 0 & \text{otherwise} \end{cases}$$

Find the marginal pdf of X_1 and X_2. Also find $P\left(X_1 \leq \frac{1}{2}\right)$ and $P(X_1 + X_2 \leq 1)$. (CO2, K3)

Or

(b) Let (X_1, X_2) have the joint cdf $F(x_1, x_2)$ and let X_1 and X_2 have the marginal cdfs $F_1(X_1)$ and $F_2(x_2)$, respectively. Then show that X_1 and X_2 are independent if and only if $F(x_1, x_2) = F_1(x_1)\, F_2(x_2)$ for all $(x_1, x_2) \in R^2$. (CO2, K3)

13. (a) Explain briefly about the hypergeometric distribution. (CO3, K4)

Or

(b) Derive the mgf of chi-square distribution. (CO3, K2)

14. (a) Derive the moments of F–distribution. (CO4, K4)

Or

(b) Let $Y_1 < Y_2 < Y_3 < Y_4$ denote the order statistics of a random sample of size 4 from a distribution having pdf $f(x) = \begin{cases} 2x & 0 < x < 1 \\ 0 & otherwise \end{cases}$. Express the pdf of Y_3 in terms of $f(x)$ and $F(x)$ and then compute $P\left(\frac{1}{2} < Y_3\right)$. (CO4, K3)

15. (a) State and prove the weak law of large numbers. (CO5, K3)

Or

(b) Suppose $X_n \xrightarrow{P} X$ and a is a constant. Then show that $aX_n \xrightarrow{P} aX$. (CO5, K3)

Part C $(5 \times 8 = 40)$

Answer **all** the questions not more than 1000 words each.

16. (a) Let X be a continuous random variable with pdf $f_x\,(x)$ and support of X. Let $Y = g(X)$, where $g(x)$ is a one-to-one differentiable function, on the support of X, S_X. Denote the inverse of g by $x = g^{-1}\,(y)$ and let $dx/dy = d\left[g^{-1}\,(y)\right]/dy$. Then show the pdf of Y is given by $f_Y\,(y) = f_x\left(g^{-1}\,(y)\right)\left|\frac{dx}{dy}\right|$, for $y \in S_Y$. where the support of Y is the set $S_Y = \{y = g(x) : x \in S(X)\}$. (CO1, K3)

Or

(b) State and prove Chebychev's inequality. (CO1, K4)

17. (a) State and prove the condition of the equvilance for the independance of two random variables.(CO2, K4)

Or

(b) Let (X_1, X_2) be a random vector such that the variance of X_2 is finite. Then show that (CO2, K3)

(i) $E[E(X_2 \mid X_1)] = E(X_2)$

(ii) $Var\,[E(X_2 | X_1) \leq Var\,(X_2)]$

18. (a) Explain in detail about the Dirichlet distribution. (CO3, K5)

Or

(b) Let $X_1, \ldots, X_n$ be independent random variables. Suppose, for $i = 1, \ldots, n$, that X_i, has a $\Gamma(\alpha_i, \beta)$ distribution. Let $Y = \sum_{i=1}^{n} X_i$. Then prove that Y has distribution. $\Gamma\left(\sum_{i=1}^{n} \alpha_i, \beta\right)$ distribution.

(CO3, K3)

19. (a) Let $Y_1 < Y_2 < \ldots < Y_n$, denote the n order statistics based on the random sample $X_1, X_2 \ldots, X_n$ from a continuous distribution with pdf $f(x)$ and support (a,b). Then derive the joint pdf of $Y_1, Y_2, \ldots, Y_n$. (CO4, K3)

Or

(b) (i) If the random variable X has a poisson distribution such that $P(X = 1) = P(X = 2)$, find $P(X = 4)$. (CO4, K3)

(ii) Derive t–distribution.

20. (a) Let T_n have a t–distribution with n degrees of freedom, $n = 1, 2, 3, \ldots$ and its cdf is $F_n(t) = \int_{-\infty}^{t} \frac{\Gamma[(n+1)/2]}{\sqrt{\pi n}\,\Gamma(n/2)} \frac{1}{\left(1 + y^2/n\right)^{(n+1)/2}} dy$, where the integrand is the pdf $f_n(y)$ of T_n. Show that T_n has a limiting standard normal distribution. (CO5, K3)

Or

(b) State and prove Central limit theorem. (CO5, K4)

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R2676

Sub. Code
511403

M.Sc. DEGREE EXAMINATION, APRIL – 2025

Fourth Semester

Mathematics

GRAPH THEORY

(CBCS – 2022 onwards)

Time : 3 Hours Maximum : 75 Marks

Part A $(10 \times 1 = 10)$

Answer **all** the questions by choosing the correct option.

1. A graph is called simple if it has (CO1, K1)

   (a) No loops

   (b) No multiple edges

   (c) Both (a) and (b)

   (d) None of the above

2. Cayley's formula states that the number of spanning trees in a complete graph K_n is (CO1, K1)

   (a) $n!$

   (b) n^{n-1}

   (c) n^n

   (d) $n(n-1)$

3. A block in a graph is (CO1, K1)

(a) A complete graph

(b) A subgraph with no cut vertices

(c) A graph with one vertex

(d) A graph with one edge

4. An Euler tour in a graph exists if and only if (CO2, K1)

(a) Every vertex has even degree

(b) All vertices have odd degree

(c) Exactly two vertices have odd degree

(d) None of the above

5. A perfect matching in a graph is (CO3, K1)

(a) A matching covering all vertices

(b) A matching covering all edges

(c) A set of edges with no common vertices

(d) None of the above

6. Ramsey's theorem deals with (CO3, K1)

(a) Chromatic number of a graph

(b) Subgraphs in edge-colored complete graphs

(c) Planarity of graphs

(d) Degree sequences of graphs

7. The chromatic number of a graph is (CO4, K1)

(a) The minimum number of colors needed to color all edges

(b) The minimum number of colors needed to color all vertices

(c) The maximum number of colors needed to color all edges

(d) None of the above

8. Vizing's theorem states that the edge chromatic number of a graph is (CO4, K2)

   (a) Equal to its maximum degree

   (b) At most one more than its maximum degree

   (c) At most two more than its maximum degree

   (d) None of the above

9. A graph is planar if it can be (CO5, K1)

   (a) Drawn without edge crossings

   (b) Drawn on a sphere

   (c) Both (a) and (b)

   (d) None of the above

10. Euler's formula for a planar graph is (CO5, K1)

   (a) $V - E + F = 2$

   (b) $V + E - F = 2$

   (c) $V - F + E = 2$

   (d) $V + F - E = 2$

Part B $(5 \times 5 = 25)$

Answer **all** the questions in not more than 500 words each.

11. (a) Prove that an edge e of G is a cut edge of G if and only if e is contained in no cycle of G. (CO1, K5)

Or

   (b) Let T be an arbitrary tree on $k+1$ vertices. Show that if G is simple and $\delta \geq k$ then G has a subgraph isomorphic to T. (CO1, K2)

12. (a) If G is a simple graph with $\upsilon \leq 3$ and $\delta \leq \upsilon / 2$, then prove that G is hamiltonian. (CO2, K5)

Or

(b) If G is non-hamiltonian simple graph with $\upsilon \geq 3$, then prove that G is degree-majorised by some $C_{m,\upsilon}$. (CO2, K2)

13. (a) Show that a matching M in G is a maximum matching if and only if G contains no M-augmenting path. (CO3, K5)

Or

(b) Prove that every 3-regular graph without cut edges has a perfect matching. (CO3, K3)

14. (a) If G is bipartite, then prove that $\chi' = \Delta$. (CO4, K4)

Or

(b) If G is k-critical then prove that $\delta \leq k-1$. (CO4, K4)

15. (a) Let G be a simple plane triangulation with $\upsilon \geq 4$. Show that G^* is a simple 2-edge- connected 3-regular planar graph. (CO5, K4)

Or

(b) Plane triangulation is a plane graph in which each face has degree three. Show that every simple plane graph is a spanning subgraph of some simple plane triangulation $\upsilon \geq 3$. (CO5, K2)

Part C $(5 \times 8 = 40)$

Answer **all** the questions not more than 1000 words each.

16. (a) Prove that a graph is bipartite if and only if contains no odd cycle. (CO1, K4)

Or

(b) Let T be an arbitrary tree on $k+1$ vertices. If G is simple and $\delta \geq k$ then prove that G has a subgraph isomorphic to T. (CO1, K5)

17. (a) Prove that a graph G with $v \geq 3$ is 2-connected if and only if any two vertices of G are connected by atleast two internally-disjoint paths. (CO2, K6)

Or

(b) Show that a nonempty connected graph is eulerian if and only if it has vertices of odd degree. (CO2, K2)

18. (a) Prove that $r(k, k) \geq 2^{k/2}$. (CO3, K3)

Or

(b) Show that G has a perfect matching if and only if $o(G-S) \leq |S|$ for all $S \subset V$. (CO3, K6)

19. (a) Let G be a k-critical graph with a 2-vertex cut $\{u, v\}$. Then prove (CO4, K4)

(i) $G = G_1 \cup G_2$ where G_1 is a $\{u, v\}$ - component of type $i (i = 1, 2)$, and

(ii) both $G_1 + uv$ and $G_2.uv$ are k-critical(where $G_2.uv$ denotes the graph obtained from G_2 by identifying u and v).

Or

(b) Show that if G is a nonempty regular simple graph with v odd, then $\chi' = \Delta + 1$. (CO4, K4)

20. (a) Prove that the following three statements are equivalent (CO5, K6)

(i) every planar graph is 4-vertex-colourable:

(ii) every plane graph is 4-face-colourable;

(iii) every simple 2-edge-connected 3-regular planar graph IS 3-edge-colourable.

Or

(b) Show that a plane triangulation G is 3-vertex colourable if and only if G is eulerian. (CO5, K5)

R2677

Sub. Code
511404

M.Sc. DEGREE EXAMINATION, APRIL – 2025

Fourth Semester

Mathematics

MEASURE AND INTEGRATION

(CBCS – 2022 onwards)

Time : 3 Hours Maximum : 75 Marks

Part A $(10 \times 1 = 10)$

Answer **all** the following objective questions by choosing the correct option.

1. Let E be any set of real numbers. Then which of following condition implies the set E is Lebesgue measurable? (CO1, K2)

   (a) $E = \bigcup_{\alpha \in A} E_\alpha$, where Λ is any indexed set and each E_α is measurable

   (b) $E = \bigcap_{\alpha \in A} E_\alpha$, where Λ is any indexed set and each E_α is measurable

   (c) $E = \mathbb{R} \sim F$, where F is any measurable set

   (d) $E = \mathbb{R} \sim F$, where F is any non-measurable set

2. Let $f : \mathbb{R} \to \mathbb{R}$ be any function. Then the function f Lebesgue measurable if (CO1, K2)

   (a) f is a simple function

   (b) $f = \chi_A$, where A any subset of $\mathbb{R}$

   (c) The range of f is a finite set

   (d) The range of f is a singleton set

3. Let $f:[0,1]\to\mathbb{R}$ be the function defined by (CO2, K2)

$$f(x)=\begin{cases} x, \ if \ x\in[0,1]\cap\mathbb{Q} \\ x^2, \ if \ x\in[0,1]\cap\mathbb{Q}^c \end{cases}$$

Then pick out the true statement.

(a) f is Lebesgue integrable on [0, 1] and $\int_0^1 f dx = 1$

(b) f is Lebesgue integrable on [0, 1] and $\int_0^1 f dx = \frac{1}{2}$

(c) f is Lebesgue integrable on [0, 1] and $\int_0^1 f dx = \frac{1}{3}$

(d) f is not Lebesgue integrable on [0, 1]

4. Let $f_n:[0,1]\to\mathbb{R}$ be the sequence of functions defined by (CO2, K2)

$$f_n(x)=\begin{cases} 2023, \ if \ 0\le x\le\frac{1}{n} \\ 2024, \ if \ \frac{1}{n}<x\le 1 \end{cases}$$

Then $\int_0^1 \lim_{n\to\infty} f_n dx =$ ________

(a) 2023

(b) 2024

(c) 0

(d) The integral does not exists

5. Let $f:(a, b) \rightarrow \mathbb{R}$ and $x \in (a, b)$. Then with respect to standard notations which of the following is NOT true? (CO3, K4)

(a) $D^{+}(-f(x)) = -D^{+}f(x)$

(b) $D^{+}(-f(x)) = -D_{+}f(x)$

(c) $D^{+}f(x) \geq D_{+}f(x)$

(d) $D^{-}f(x) \geq D_{-}f(x)$

6. Let (a, b) is a finite interval and $f \in L(a,b)$. Suppose E denotes the Lebesgue set of f. Then which of the following is necessarily true? (CO3, K4)

(a) $\lim_{h\to 0} \frac{1}{h}\int_0^h |f(x+t) - f(x)| dt \neq 0$ for all $x \in E$

(b) $\lim_{h\to 0} \frac{1}{h}\int_0^h |f(x+t) - f(x)| dt \neq 0$ for some $x \in E$

(c) $\lim_{h\to 0} \frac{1}{h}\int_0^h |f(x+t) - f(x)| dt = 0$ for all $x \in E$

(d) $\lim_{h\to 0} \frac{1}{h}\int_0^h |f(x+t) - f(x)| dt \neq 0$ for all $x \in E^c$

7. Let v_1 and v_2 be two measures on a measurable space $[X, S]$. Then v_1 is mutually singular to v_2 provided (CO4, K3)

(a) $v_1(A) = 0, v_2(A) = 0$ for some $A \in S$

(b) $v_1(A) = 0, v_2(A^c) \neq 0$ for some $A \in S$

(c) $v_1(A) \neq 0, v_2(A^c) = 0$ for some $A \in S$

(d) $v_1(A) = 0, v_2(A^c) = 0$ for some $A \in S$

8. Let $[X, S, \mu]$ be a measure space and let $f: X \to \mathbb{R}$ be any non-negative function such that $\int_X f\, d\mu$ exists. Suppose v the set function on S defined by $v(E) = \int_E f d\mu$ for all $E \in S$. Then which of the following is necessarily true? (CO4, K3)

(a) v is measure on S such that $v << \mu$

(b) v is measure on S such that $\mu << v$

(c) v is measure on S such that $v \perp \mu$

(d) v is not a measure on S

9. Let $[X, S, \mu]$ and $[Y, \mathcal{J}, v]$ be finite measure spaces. For $V \in S \times \mathcal{J}$ define $\phi(x) = v(V_x)$, $\psi(y) = v(V^y)$ for each $x \in X$, $x \in Y$. Then which of the following is NOT true?

(CO5, K4)

(a) ϕ is $\mathcal{J}$-measurable, ψ is S-measurable and $\int_X \phi d\mu = \int_Y \psi dv$

(b) ϕ is S-measurable, ψ is $\mathcal{J}$-measurable and $\int_X \phi d\mu = \int_Y \psi dv$

(c) ϕ is S-measurable, ψ is $\mathcal{J}$-measurable and $\int_X \phi d\mu \neq \int_Y \psi dv$

(d) ϕ is $\mathcal{J}$-measurable, ψ is S-measurable and $\int_X \phi d\mu \neq \int_Y \psi dv$

10. Consider $f(x, y) = x^2 y$ for all $(x, y) \in \mathbb{R}^2$. If $E = \{(x, y) \in \mathbb{R}^2 \mid 0 \le x \le 3, \ 1 \le y \le 2\}$ then which of the following is true? (CO5, K4)

(a) $\iint_E f(x, y)\,dx\,dy = \frac{22}{7}$

(b) $\iint_E f(x, y)\,dx\,dy = \frac{27}{7}$

(c) $\iint_E f(x, y)\,dx\,dy = \frac{27}{3}$

(d) $\iint_E f(x, y)\,dx\,dy = \frac{27}{2}$

Part B $(5 \times 5 = 25)$

Answer **all** the questions not more than 500 words each.

11. (a) Let $\mathcal{M}$ denotes the collection of all Lebesgue measurable sets. Prove that $\mathcal{M}$ forms a $\sigma-$algebra. (CO1, K2)

Or

(b) Let μ be a measure on a ring $\mathcal{R}$. Then prove that the set function μ^* on $\mathcal{H}(\mathcal{R})$defined by defined by

$\mu^*(E) = \inf\left\{\sum_{n=1}^{\infty} \mu(E_n) \mid E \subseteq \bigcup_{n=1}^{\infty} E_n,\ E_n \in \mathcal{R}\right\}$ is an outer measure of $\mathcal{H}(\mathcal{R})$. (CO1, K2)

12. (a) State Monotone Convergence theorem and sketch a proof of it. (CO2, K2)

Or

(b) Let f and g be non-negative measurable functions. Then prove that $\int (f+g)dx = fdx + \int gdx$. (CO2, K2)

13. (a) Let E be a set of measure zero. Then show that there exists a function defined on $\mathbb{R}$ which is continuous and increasing everywhere and for which each derivate is infinite at each point of E. (CO3, K4)

Or

(b) Evaluate at $x=0$ the four derivates of the continuous function

$$f(x)=\begin{cases} ax\sin^2\frac{1}{x}+bx\cos^2\frac{1}{x}, if\ x>0 \\ 0, \qquad if\ x=0 \\ a'x\sin^2\frac{1}{x}+b'x\cos^2\frac{1}{x},\ if\ x<0 \end{cases}$$

where $a<b$ and $a'<b'$. (CO3, K4)

14. (a) Prove that countable union of sets positive with respect to a signed measure v is a positive set. (CO4, K3)

Or

(b) Let $[X,S,\mu]$ be a measure space and let $\int f\,d\mu$ exist. Define v by $v(E)=\int_E fdv$ for $E\in S$. Then find a Jordan decomposition of v. (CO4, K3)

15. (a) With respect to standard notations, if $E \in S \times \mathcal{J}$ then for each $x \in X$ and $y \in Y$ then prove that $E_x \in \mathcal{J}$ and $E^y \in S$. (CO5, K4)

Or

(b) Let f be non-negative $S \times \mathcal{J}$-measurable function and let $\phi(x) = \int_Y f_x dv$, $\psi(y) = \int_X f^y d\mu$ for each $x \in X$, $y \in Y$. Then prove that ϕ is S-measurable, ψ is $\mathcal{J}$-measurable and $\int_X \phi d\mu = \int_{X \times Y} fd(\mu \times v) = \int_Y \psi dv$. (CO5, K4)

Part C (5 × 8 = 40)

Answer **all** the questions not more than 1,000 words each.

16. (a) Let E be any set of real numbers such that $m^*(E) < \infty$. Then sketch a proof that E is measurable if and only if for each $\varepsilon > 0$ there exist disjoint bounded open intervals $\{I_i\}_{i=1}^n$ such that $m^*\left(E \Delta \left(\bigcup_{i=1}^n I_i\right)\right) < \varepsilon$. (CO1, K5)

Or

(b) Prove that the outer measure of an interval equals its length. (CO1, K3)

17. (a) Let $f : [a,b] \to \mathbb{R}$ be a bounded function. If f is Riemann integrable then prove that f is Lebesgue integrable and $\mathcal{R}\int_a^b fdx = \int_a^b fdx$. (CO2, K3)

Or

(b) State Dominated Convergence Theorem and sketch a proof of it. (CO2, K2)

18. (a) If f is a finite valued monotone increasing function defined on a finite interval $[a,b]$ then prove that f' is measurable and $\int_a^b f'dx \le f(b) - f(a)$. (CO3, K4)

Or

(b) If $f \in L(a,b)$ where (a,b) is a finite interval, then show that there exists a set $E \subseteq (a,b)$ such that $m^*([a,b] \sim E) = 0$ and $\lim_{h\to 0} \frac{1}{h} \int_x^{x+h} |f(t) - \xi| dt = |f(t) - \xi|$ for all real ξ and all $x \in E$. (CO3, K5)

19. (a) Let $[X, S, \mu]$ be a finite measure space and v be finite measure on S such that $v << \mu$. Then prove that there exists a finite valued non-negative measurable function f on X such that $v(E) = \int_E f d\mu$ for all $E \in S$. (CO4, K3)

Or

(b) State and prove Hahn's decomposition theorem. (CO4, K3)

20. (a) Let $[X, S, \mu]$ and $[Y, \mathcal{J}, v]$ be finite measure spaces. For $V \in S \times \mathcal{J}$ define $\phi(x) = v(V_x)$, $\psi(y) = v(V^y)$ for each $x \in X$, $y \in Y$. Then prove that ϕ is S-measurable ψ is $\mathcal{J}$-measurable and $\int_X \phi \, d\mu = \int_Y \psi \, dv$. (CO5, K4)

Or

(b) If $\mathcal{A}$ is an algebra then prove that the σ-algebra generated by $\mathcal{A}$ is same as the smallest monotone class containing $\mathcal{A}$. (CO5, K4)

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